

Unit [IV]

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ques (31) Let (X, d) and (Y, ρ) be two metric spaces and let $f: X \rightarrow Y$ be a function. Then prove that the function f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y .

ques (32) Let (X, d) and (Y, ρ) be two metric spaces and let $f: X \rightarrow Y$ be a function. Then prove that $f^{-1}(F)$ is closed in X whenever F is closed in Y .

ques (33) Define Compact metric space.

Prove that a closed subset of a compact metric space is compact.

ques (34) Prove that every compact metric space has the Bolzano-Weierstrass property.

ques (35) Prove that a compact subset of a metric space is closed and bounded.

ques (36) Define finite intersection property. Prove that a metric space (X, d) is compact if and only if every collection of closed subsets of X having finite

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finite intersection property has a non-empty intersection.

ques 37) Let (X, d) be a metric space and let $\{A_\alpha : \alpha \in \Lambda\}$ be a family of connected sets in X such that $\bigcap_{\alpha \in \Lambda} A_\alpha \neq \emptyset$. Then prove that $\bigcup_{\alpha \in \Lambda} A_\alpha$ is connected.

ques 38) Define connected and disconnected spaces

ques 39) Prove that every continuous image of a connected set is also connected.

ques 40) Prove that a subset $A \subseteq \mathbb{R}$ in a usual metric space \mathbb{R}_u is connected if and only if it is an interval.

Unit [V]

ques 41) Prove that continuity is necessary condition but not a sufficient condition for the existence of a finite derivative for the function of a complex variable

ques 42) Define cross ratio. Prove that cross-ratios are invariant under a bilinear transformation