B. Sc-II year
Mathematics

Paper - Advanced Calculus
Important questions of annual exam.

UNIT-I

Q-① Every Convergent Sequence is bounded, prove.

Q-② prove that every Cauchy sequence is bounded but the Converse is not true.

Q-③ prove that the infinite series

$$\Sigma \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$$

is Convergent if $p > 1$ and divergent if $p \leq 1$.

Q-④ Test for Convergence of the following series

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \cdots$$

Q-⑤ find whether the series

$$x + \frac{2^2 x^2}{\lfloor 2} + \frac{3^3 x^3}{\lfloor 3} + \frac{4^4 x^4}{\lfloor 4} + \frac{5^5 x^5}{\lfloor 5} + \cdots, x > 0$$

is Convergent or divergent.

UNIT-II

Q-① Test the following function for Continuity at $x=0$

$$f(x) = \begin{cases} x.\sin\frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

also show that it is Continuous for all values of x.

2) Test for Continuity of the following function at $x=0$

$$f(x) = \frac{1}{1-e^{1/x}}, \quad x \neq 0 \text{ and } f(0)=0.$$

3) Show that the following function

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

is Continuous and differentiable at $x=0$.

4) Verify Rolle's theorem in the interval $[2,4]$ for the function $f(x) = x^2 - 6x + 8$.

5) Verify Lagrange's mean value theorem for the function $f(x) = \sqrt{x^2-4}$ in the Interval $[2,4]$.

UNIT-III

1) Examine the Continuity of $f(x,y)$ given below at the point $(0,0)$

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

2) If $u = e^{xyz}$ then show that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$$

3) If $x^x y^y z^z = c$ then show that

$$\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1} \text{ when } x=y=z.$$

Q④ If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u.$$

Q⑤ Transform the equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

by putting $x = \frac{1}{z}$.

UNIT-IV

Q① Find the equation of the evolute of the parabola $y^2 = 4ax$.

Q② ① Discuss the maximum or minimum values the function $u = x^3+y^3-3axy$.

② Discuss the maximum or minimum value of

$$u = x^3y^2(1-x-y)$$

Q③ Prove that.

① $\Gamma m \, \Gamma m+\frac{1}{2} = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma 2m$

where m is a positive real number.

② $\int_a^b (x-a)^{m-1}(b-x)^{n-1}\,dx = (b-a)^{m+n-1} \cdot B(m,n)$

where $m>0$ and $n>0$.

Q④ Find the minimum value of the function $u = x^2+y^2+z^2$ having given $ax+by+cz=p$

Q⑤ Find the maxima and minima of the follo[wing] function $u = \sin x \cdot \sin y \cdot \sin(x+y)$

UNIT-V

Q-① Evaluate ① $\int_0^1 \int_0^1 \frac{dx\,dy}{\sqrt{1-x^2}\,\sqrt{1-y^2}}$

② $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx\,dy}{1+x^2+y^2}$

Q-② Evaluate $\iint_R xy\,dx\,dy$ over the region in positive quadrant for which $x+y \leq 1$.

Q-③ Find the value of $\iint_R x^2y^2\,dx\,dy$ where R region $x^2+y^2 \leq 1$.

Q-④ Evaluate $\iint r^2\,d\theta\,dr$ over the region of the circle $r = a\cos\theta$.

Q-⑤ Find the volume of the solid generated the revolution of the circle $x^2+y^2 = a^2$ about x-axis.

———x———x———

Note- For solutions, please see class notes.