

B. Sc - II year
Mathematics

Paper - Advanced Calculus
Important questions of annual exam.

UNIT - I

Q-① Every Convergent sequence is bounded, prove.

Q-② prove that every Cauchy sequence is bounded but the converse is not true.

Q-③ prove that the infinite series
$$\sum \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$$

is convergent if $p > 1$ and divergent if $p \leq 1$.

Q-④ Test for convergence of the following series

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$$

Q-⑤ Find whether the series

$$x + \frac{2^2 x^2}{2} + \frac{3^3 x^3}{3} + \frac{4^4 x^4}{4} + \frac{5^5 x^5}{5} + \dots, x > 0$$

is convergent or divergent.

UNIT - II

Q-① Test the following function for continuity at $x=0$

$$f(x) = \begin{cases} x \sin \frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

also show that it is continuous for all values of x .

3② Test for Continuity of the following function at $x=0$

$$f(x) = \frac{1}{1 - e^{1/x}}, \quad x \neq 0, \text{ and } f(0) = 0.$$

3③ show that the following function

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

is continuous and differentiable at $x=0$.

4④ verify Rolle's theorem in the interval $[2, 4]$ for the function $f(x) = x^2 - 6x + 8$.

4⑤ Verify Lagrange's mean value theorem for the function $f(x) = \sqrt{x^2 - 4}$ in the interval $[2, 4]$.

UNIT-III

1① Examine the continuity of $f(x, y)$ given below at the point $(0, 0)$

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

2② If $u = e^{xyz}$ then show that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^3) e^{xyz}$$

3③ If $x^x y^y z^z = c$ then show that

$$\frac{\partial^2 c}{\partial x \partial y} = -(x \cdot \log ex)^{-1} \text{ when } x = y = z.$$

Q4) If $u = \sin^{-1} \left(\frac{x^2 + y^2}{x + y} \right)$ then show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u.$$

Q5) Transform the equation

$$x^2 \frac{dy^2}{dx^2} + x \frac{dy}{dx} + y = 0$$

by putting $x = \frac{1}{z}$.

UNIT-IV

Q1) find the equation of the evolute of the parabola $y^2 = 4ax$.

Q2) ① Discuss the maximum or minimum values the function $u = x^3 + y^3 - 3axy$.

② Discuss the maximum or minimum value of $u = x^3 y^2 (1 - x - y)$

Q3) prove that

$$\Gamma(m) \Gamma(m + \frac{1}{2}) = \frac{\sqrt{\pi}}{2^{2m+1}} \Gamma(2m)$$

where m is a positive real number.

$$\int_a^b (x-a)^{m-1} (b-x)^{n-1} dx = (b-a)^{m+n-1} \cdot B(m, n)$$

where $m > 0$ and $n > 0$.

Q4) find the minimum value of the function $u = x^2 + y^2 + z^2$ having given $ax + by + cz = f$

Q5) find the maxima and minima of the following function $u = \sin x, \sin y, \sin(x+y)$

UNIT - V

Q-1 Evaluate ① $\int_0^1 \int_0^1 \frac{dx dy}{\sqrt{1-x^2} \sqrt{1-y^2}}$

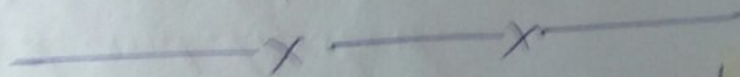
② $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx dy}{1+x^2+y^2}$

Q-2 Evaluate $\iint_R xy \, dx dy$ over the region in positive quadrant for which $x+y \leq 1$.

Q-3 find the value of $\iint_R x^2 y^2 \, dx dy$ where R region $x^2 + y^2 \leq 1$.

Q-4 Evaluate $\iint r^2 \, d\theta dr$ over the region of circle $r = 2 \cos \theta$.

Q-5 find the volume of the solid generated the revolution of the circle $x^2 + y^2 = a^2$ about x -axis.



Note - for solutions, please see class notes.