B. Sc IIIrd year : Mathematics

Paper II : Real and Complex Analysis

Important questions

Unit [I]

ques (1) Define lower and upper Riemann Sums

ques (2) If $f : [a, b] \to R$ is bounded function then for any partition P of $[a, b]$, $L(P, f)$ and $U(P, f)$ are bounded

ques (3) Define lower and upper Riemann integrals

ques (4) State and prove Riemann's Criterion for integrability

ques (5) Prove that every Continuous function is Riemann integrable

ques (6) Prove that every monotonic function is Riemann integrable

ques (7) If $f \in R[a, b]$, then prove that $|f| \in R[a, b]$ and $\left|\int_a^b f\,dx\right| \leq \int_a^b |f|\,dx$

ques (8) State and prove fundamental theorem of integral Calculus

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Ques 16 If $F(t) = \int_a^b f(x,t)\,\phi(x)\,dx$ where f and f_t are continuous in $a \le x \le b$, $\alpha \le t \le \beta$ and $\phi(x)$ is bounded and integrable on $[a,b]$, then $F'(t)$ exists and $F'(t) = \int_a^b \frac{\partial f(x,t)}{\partial t}\,\phi(x)\,dx$ on $[\alpha,\beta]$

Ques 17 If $|\alpha| < 1$, then show that

$$\int_0^{\pi} \frac{\log(1+\alpha\cos x)}{\cos x}\,dx = \pi \sin^{-1}\alpha$$

Ques 18 Show that

$$F(\alpha) = \int_0^{\pi} \frac{\log(1+\cos\alpha \sin x)}{\sin x}\,dx = \frac{\pi^2 - 4\alpha^2}{4}$$

for all $\alpha \in (-\pi, \pi)$

Ques 19 Find the Fourier Series of the function $f(x) = x^2$ in the interval $-\pi < x < \pi$. Hence deduce that

$$\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$

Ques 20 Find the Fourier series of the function $f(x) = x \sin x$ in the interval $-\pi < x < \pi$

Unit [III]

Ques (21) Define Metric Space. Let a mapping $d: R \times R \to R$ be defined as follows:

$$d(x,y) = \frac{|x-y|}{1+|x-y|} \quad \forall\, x, y \in R$$

Then prove that d is a metric on R.

Ques (22) Define bounded metric space. Let (X, d) be any metric space and let M be a positive real number, then there exists a metric d^* for X such that the metric space (X, d^*) is bounded with $\delta(X) \leq M$

Ques (23) Define open sphere and open set in a metric space.

In a metric space prove that every open sphere is an open set

Ques (24) In a metric space prove that, the intersection of a finite number of open sets is also an open set

Ques (25) Define closed sphere and closed set in a metric space. In a metric space prove that every closed sphere is a closed set.

Ques 26. In a metric space, prove that the union of a finite number of closed sets is also a closed set.

Ques 27 Define Cauchy sequence in a metric space. Prove that every convergent sequence in a metric space is a Cauchy sequence.

Ques 28 Define complete metric space. Let (X, d) be a complete metric space and let (Y, d) be a subspace of (X, d). Then prove that Y is complete if and only if Y is closed.

Ques 29 Define first category and second category sets. Prove that any complete metric space is a set of the second category

Ques 30 Define first countable and second countable spaces. Prove that every metric space is first countable

12/6/2020.